AD-A066 853 NAVAL UNDERSEA RESEARCH AND DEVELOPMENT CENTER SAN D--ETC F/G 17/1
ON THE FREQUENCY SCALING OF SONAR TRANSDUCER ARRAYS, (U)
JUL 71 D L CARSON, H H DING, J A WARD
UNCLASSIFIED NUC-TN-576 NL

1 OF 1
ADA
066853

END
DATE
FILMED
6-79
DDC

MOST Project -4

(14) NUC-TN-576

LEVEL II

001587

Good B-5.

1 8

NAVAL UNDERSEA RESEARCH AND DEVELOPMENT CENTER

AD A0 66853

(6) ON THE FREQUENCY SCALING OF SONAR TRANSDUCER ARRAYS

by

(10) D. L. Carson & H. H. Ding, Code 601
J. A. Ward, Code 605

Sponsored by NAVSHIPS Code 00V12

(12) 12p.

(11) July 1971

San Diego, California

~~FOR OFFICIAL USE ONLY~~

001587 Hg-15

404 762 LB

INTRODUCTION

f sub o

It is well established that given the computations for predicting the performance of a transducer array at a frequency band centered at f_o one can easily scale the entire problem to a new frequency band centered at f_s. The purpose of this note is simply to document with simple examples a recipe for performing such a frequency scaling of the entire problem and thereby help avoid wasting time re-thinking this procedure whenever it is needed. No attempt will be made to treat the subject in general or in depth.

f sub s

CERAMIC RING EQUATIONS

For a single piece of 33 mode longitudinally-poled piezoelectric ceramic, the following equations can be shown to apply:

$$\begin{bmatrix} F_1 \\ F_2 \\ E \end{bmatrix} = \begin{bmatrix} Z^C_{11} & Z^C_{12} & Z^C_{13} \\ -Z^C_{12} & -Z^C_{11} & Z^C_{13} \\ Z^C_{13} & -Z^C_{13} & Z^C_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix} \stackrel{d}{=} \begin{bmatrix} Z^C \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix} \tag{1.1}$$

where: F_1 = force on the right side.

F_2 = " " " left " .

v_1 = velocity on the right side.

v_2 = " " " left " .

E = voltage across the piece.

I = current in the piece.

$[Z^C]$ = 3 X 3 transfer matrix which describes the piece as a 3-port network.

From MCR #1 (among others) we may write:

$$Z^C_{11} = -\hat{j}\rho c A \cot\theta \tag{1.2}$$

$$Z^C_{12} = \hat{j}\rho c A \csc\theta \tag{1.3}$$

$$Z^C_{13} = \frac{-\hat{j}\rho c \ell G33}{\theta} \tag{1.4}$$

$$Z^C_{33} = \frac{-\hat{j}\rho c \ell^2 \beta^{LC}_{33} S33D}{\theta A} \tag{1.5}$$

Where: ρ = ceramic density (Kg/m^3)

c = ceramic sound velocity = $c_{r_e}(1 - \hat{j}c_m)$ (m/sec)

$\theta = k\ell = \frac{\omega \ell}{c}$ (radians), $\omega = 2\pi f$

ℓ = ceramic length (per piece) (m.)

A = ceramic area (m^2)

G33, $\beta^{LC}_{33} = \frac{1}{E33T}$, $S33D = \frac{1}{\rho c^2}$ (ceramic parameters)

"M" $\Rightarrow$ loss multiplier

$E33T = E33T_{re}\,(1 - \hat{j}E33TM)$

$S33D = S33D_{re}\,(1 - \hat{j}S33DM)$

$G33 = G33_{re}\,(1 - \hat{j}G33M)$

CERAMIC STACK SCALING EQUATIONS

For a cylinder consisting of N identical rings of ceramic, we may write:

$$\begin{bmatrix} F_1 \\ F_2 \\ E \end{bmatrix} = \begin{bmatrix} Z^{CN}_{11} & Z^{CN}_{12} & Z^{CN}_{13} \\ -Z^{CN}_{12} & -Z^{CN}_{11} & Z^{CN}_{13} \\ Z^{CN}_{13} & -Z^{CN}_{13} & Z^{CN}_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix} \underline{\underline{d}} \begin{bmatrix} Z^{CN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix} \tag{2.1}$$

Where: F_1, v_1 are on the right side of the stack.

F_2, v_2 " " " left " " " " .

E, I are at the electrical port.

$[Z^{CN}]$ is the 3-port description of the stack.

Again, from MART contract, Reference #1:

$$Z^{CN}_{11} = \frac{-\hat{j}\rho cA}{N\theta}\left[N\left(\frac{\theta}{\sin\theta} - \frac{(G33)^2}{S33D\beta^{LC}_{33}}\right)\sinh\alpha\coth N\alpha - \frac{(G33)^2}{S33D\beta^{LC}_{33}}\right] \tag{2.2}$$

$$Z^{CN}_{12} = \frac{\hat{j}\rho cA}{N\theta}\left[N\left(\frac{\theta}{\sin\theta} - \frac{(G33)^2}{S33D\beta^{LC}_{33}}\right)\sinh\alpha\operatorname{csc} N\alpha - \frac{(G33)^2}{S33D\beta^{LC}_{33}}\right] \tag{2.3}$$

$$Z^{CN}_{13} = \frac{-\hat{j}\rho c\ell(G33)}{N\theta} \tag{2.4}$$

$$Z^{CN}_{33} = \frac{-\hat{j}\rho c\ell^2(S33D\beta^{LC}_{33})}{AN\theta} \tag{2.5}$$

$$\text{where:}\quad \alpha = \cosh^{-1}\left[\frac{(G33)^2\sin\theta - (S33D\beta^{LC}_{33})\theta\cos\theta}{(G33)^2\sin\theta - (S33D\beta^{LC}_{33})\theta}\right] \tag{2.5.1}$$

Rewriting the above equations to display area and length dependence, we have:

$$Z^{CN}_{11} = K^{CN}_{11}A \tag{2.6}$$

$$Z^{CN}_{12} = K^{CN}_{12}A \tag{2.7}$$

$$Z^{CN}_{13} = K^{CN}_{13}\ell \tag{2.8}$$

$$Z^{CN}_{33} = \frac{K^{CN}_{33}\ell^2}{A} \tag{2.9}$$

where: K_{ij}^{CN} is a function of $\theta = \frac{\omega \ell}{c}$, $\omega = 2\pi f$, f = frequency.

Thus, if we assume $\omega\ell$ to be invariant in our scaling, we know K_{ij}^{CN} to be invariant.

Knowing the 3 X 3 representation of a ceramic cylinder, we can perform the following manipulation:

$$\begin{bmatrix} F_1^0 \\ F_2^0 \\ E^0 \end{bmatrix} = \begin{bmatrix} K_{11}^{CNO}A_0, & K_{12}^{CNO}A_0, & K_{13}^{CNO}\ell_0 \\ -K_{12}^{CNO}A_0, & -K_{11}^{CNO}A_0, & K_{13}^{CNO}\ell_0 \\ K_{13}^{CNO}\ell_0, & -K_{13}^{CNO}\ell_0, & K_{33}^{CNO}\ell_0^2/A_0 \end{bmatrix} \begin{bmatrix} v_1^0 \\ v_2^0 \\ I^0 \end{bmatrix} \tag{3.1}$$

Let: A_0 = ceramic area, ℓ_0 = ceramic piece length and ω_0 = reference operating frequency.

Define: $\frac{1}{s} = \frac{\omega_0}{\omega_s}$ where ω_s = scaled operating frequency (3.2)

$= s \times \omega_0$

Thus, since $\omega_0\ell_0 = \omega_s\ell_s$ = invarient; then $\frac{\ell_0}{\ell_s} = s$ (3.3)

and $\frac{A_0}{A_s} = (\frac{\ell_0}{\ell_s})^2 = s^2$ (3.4)

where: ℓ_s = length after scaling for operation at ω_s

A_s = area " " " " " "

For (3.1) above, we now have (after scaling):

$$\begin{bmatrix} F_1^s \\ F_2^s \\ E^s \end{bmatrix} = \begin{bmatrix} K_{11}^{CNO}A_0/s^2, & K_{12}^{CNO}A_0/s^2, & K_{13}^{CNO}\ell_0/s \\ -K_{12}^{CNO}A_0/s^2, & -K_{11}^{CNO}A_0/s^2, & K_{13}^{CNO}\ell_0/s \\ K_{13}^{CNO}\ell_0/s, & -K_{13}^{CNO}\ell_0/s, & K_{33}^{CNO}\ell_0^2/A_0 \end{bmatrix} \begin{bmatrix} v_1^s \\ v_2^s \\ I^s \end{bmatrix} \tag{3.5}$$

With the foregoing in mind, we may choose a ceramic stack designed for operation at ω_0, with length ℓ_0 and area A_0 which is described by equation (3.1). If we now wish to scale this stack for operation at any other frequency, ω_s, we may ascribe a new piece length, ℓ_s, and a new area, A_s, and then write the description of the scaled stack as follows:

$$\begin{bmatrix} F_1^s \\ \\ F_2^s \\ \\ E^s \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{11}^{CNO}}{s^2} & \dfrac{Z_{12}^{CNO}}{s^2} & \dfrac{Z_{13}^{CNO}}{s} \\ \\ \dfrac{-Z_{12}^{CNO}}{s^2} & \dfrac{-Z_{11}^{CNO}}{s^2} & \dfrac{Z_{13}^{CNO}}{s} \\ \\ \dfrac{Z_{12}^{CNO}}{s} & \dfrac{-Z_{13}^{CNO}}{s} & Z_{33}^{CNO} \end{bmatrix} \begin{bmatrix} v_1^s \\ \\ v_2^s \\ \\ I^s \end{bmatrix} \tag{3.6}$$

It is advantageous to remove the scaling factor, s, from $[Z^{CN}]$ making $[Z^{CN}]$ invariant during scaling, (i.e. $[Z^{CN}] \rightarrow [Z^{CNO}]$ for ω_o).

From (3.6):
$$\begin{bmatrix} F_1^o \\ F_2^o \\ E^o \end{bmatrix} = \begin{bmatrix} sF_1^s \\ sF_2^s \\ E^s \end{bmatrix} = \begin{bmatrix} Z_{11}^{CNO} & Z_{12}^{CNO} & Z_{13}^{CNO} \\ -Z_{12}^{CNO} & -Z_{11}^{CNO} & Z_{13}^{CNO} \\ Z_{13}^{CNO} & -Z_{13}^{CNO} & Z_{33}^{CNO} \end{bmatrix} \begin{bmatrix} v_1^s/s \\ v_2^s/s \\ I^s \end{bmatrix} \tag{4.1}$$

$$= [Z^{CNO}] \begin{bmatrix} v_1^s/s \\ \\ v_2^s/s \\ \\ I^s \end{bmatrix} = [Z^{CNO}] \begin{bmatrix} v_1^o \\ \\ v_2^o \\ \\ I^o \end{bmatrix}$$

NON-PIEZOELECTRIC SCALING EQUATIONS

Similarly, it is shown in MCR #1 that the 2-port description of any straight section can be written in the form:

$$\begin{bmatrix} F_1^o \\ \\ F_2^o \end{bmatrix} = \begin{bmatrix} K_{11}^o A^o & K_{12}^o A^o \\ \\ -K_{12}^o A^o & K_{22}^o A^o \end{bmatrix} \begin{bmatrix} v_1^o \\ \\ v_2^o \end{bmatrix} \stackrel{d}{=} \begin{bmatrix} Z^o \end{bmatrix} \begin{bmatrix} v_1^o \\ \\ v_2^o \end{bmatrix} \tag{4.2}$$

where: the subscript 1 implies left side
" " 2 " right side

A^o = straight section area.
K_{ij}^o is a function of $\theta = \frac{\omega \ell}{c}$

c = sound velocity in the straight section
ω = frequency (radians/sec)
ℓ = straight section length.

By using reasoning similar to that accompanying equations (3.2), (3.3), and (3.4), we can write:

$$\begin{bmatrix} F_1^s \\ F_2^s \end{bmatrix} = \frac{A_o}{s^2} \begin{bmatrix} K_{11}^o & K_{12}^o \\ -K_{12}^o & K_{22}^o \end{bmatrix} \begin{bmatrix} v_1^s \\ v_2^s \end{bmatrix} = \frac{1}{s^2} \begin{bmatrix} Z_{11}^o & Z_{12}^o \\ -Z_{12}^o & Z_{22}^o \end{bmatrix} \begin{bmatrix} v_1^s \\ v_2^s \end{bmatrix} \qquad (4.3)$$

where $[Z_{ij}^o]$ are valid at operating frequency ω_o.

Therefore:

$$\begin{bmatrix} sF_1^s \\ sF_2^s \end{bmatrix} = [Z_{ij}^o] \begin{bmatrix} v_1^s/s \\ v_2^s/s \end{bmatrix} \quad (i,j = 1,2) \qquad (4.4)$$

where $[Z_{ij}^c]$ is invariant during scaling to other operating frequencies.

It can be easily be shown that, for a conical section, equation (4.4) is valid in form.

We now are in a position to examine an entire transducer element made up of many two- and three-port networks of the form shown in equations (4.1) and (4.4). Since the transfer matrices in these equations are functions only of θ and θ is held constant when scaling to a new operating frequency by assumption, any conceivable combination of these matrices, such as might be found in a complex sonar element must also be independent of s.

e.g:

$$\begin{bmatrix} sF_r^s \\ sF_T^s \\ E^s \end{bmatrix} = \begin{bmatrix} Z_{11}^o & Z_{12}^o & Z_{13}^o \\ Z_{21}^o & Z_{22}^o & Z_{23}^o \\ Z_{31}^o & Z_{32}^o & Z_{33}^o \end{bmatrix} \begin{bmatrix} v_r^s/s \\ v_T^s/s \\ I^s \end{bmatrix} \qquad (5.1)$$

For a single element where $[Z_{ij}^o]$ is invariant during scaling.

The radiation impedance for a simple circular piston can be written in the form:

$$F_r^o = \rho c A_o v_r^o [R^o(\theta) + jX^o(\theta)] \tag{5.2}$$

where: F_r^o = radiating face force on the water

v_r^o = " " " velocity

ρ = density of sea water

c = sound velocity of sea water

A_o = radiating face area = πa^2

a = radius of the radiating face

θ = $ka = \frac{\omega a}{c}$

ω = frequency

$R^o(\theta)$ and $X^o(\theta)$ are functions relating array geometry, etc.

Similarly to the steps leading to equations (4.1) and (4.4),

$$F_r^s = \rho c \frac{A_o}{s^2} v_r^s [R^o(\theta) + jX^o(\theta)] \tag{5.3}$$

or

$$sF_r^s = A_o \rho c \left(\frac{v_r^s}{s}\right) [R^o(\theta) + jX^o(\theta)] = K^o\left(\frac{v_r^s}{s}\right) \tag{6.1}$$

where: K^o is invariant during scaling provided that ωa is held fixed.

Returning to equation (5.1) and shorting out the tail port by terminating with air (set $F_T \equiv 0$):

$$\begin{bmatrix} sF_r^s \\ 0 \\ E^s \end{bmatrix} = [Z^{CTO}] \begin{bmatrix} v_r^s/s \\ v_T^s/s \\ I^s \end{bmatrix} \tag{6.2}$$

$$\begin{bmatrix} sF_r^s \\ E^s \end{bmatrix} = \begin{bmatrix} Z_{11}^{TO} & Z_{12}^{TO} \\ Z_{21}^{TO} & Z_{22}^{TO} \end{bmatrix} \begin{bmatrix} v_{r/s}^s \\ I^s \end{bmatrix} = \begin{bmatrix} Z^{TO} \end{bmatrix} \begin{bmatrix} v_{r/s}^s \\ I^s \end{bmatrix} \qquad (6.3)$$

($[Z^{TO}]$ invariant during scaling.)

Converting to A-form:

$$\begin{bmatrix} E^o \\ I^o \end{bmatrix} = \begin{bmatrix} E^s \\ I^s \end{bmatrix} = \begin{bmatrix} A^o & B^o \\ C^o & D^o \end{bmatrix} \begin{bmatrix} F_r^o \\ v_r^o \end{bmatrix} = \begin{bmatrix} A^o & B^o \\ C^o & D^o \end{bmatrix} \begin{bmatrix} sF^s \\ v_{r/s}^s \end{bmatrix} \qquad (6.4)$$

where: $\begin{bmatrix} A^o & B^o \\ C^o & D^o \end{bmatrix}$ is invariant during scaling.

This (6.4) is the familiar 2X2 representation of a transducer relating the variables at the electrical port to the variables at the head mechanical port. One further note on equation (6.3):

$$Z_{11}^{TO} = \frac{1}{Z_{22}^{CTO}} (Z_{11}^{CTO} Z_{22}^{CTO} - Z_{12}^{CTO} Z_{21}^{CTO}) \qquad (6.5)$$

$$Z_{12}^{TO} = \frac{1}{Z_{22}^{CTO}} (Z_{13}^{CTO} Z_{22}^{CTO} - Z_{12}^{CTO} Z_{23}^{CTO}) \qquad (6.6)$$

$$Z_{21}^{TO} = \frac{1}{Z_{22}^{CTO}} (Z_{31}^{CTO} Z_{22}^{CTO} - Z_{21}^{CTO} Z_{32}^{CTO}) \qquad (6.7)$$

$$Z_{22}^{TO} = \frac{1}{Z_{22}^{CTO}} (Z_{22}^{CTO} Z_{33}^{CTO} - Z_{23}^{CTO} Z_{32}^{CTO}) \qquad (6.8)$$

As can be seen, invariance is indeed preserved when the tail port is removed. Continuing with equation (6.4) and substituting with equation (6.1):

$$\begin{bmatrix} E^s \\ I^s \end{bmatrix} = \begin{bmatrix} A^o & B^o \\ C^o & D^o \end{bmatrix} \begin{bmatrix} K^o v_r^s/s \\ v_r^s/s \end{bmatrix} = \begin{bmatrix} (A^o K^o) + B^o \\ (C^o K^o) + D^o \end{bmatrix} \begin{bmatrix} v_r^s/s \\ v_r^s/s \end{bmatrix} \qquad (7.1)$$

CONCLUSIONS

With this final result in hand, we make the following comments and summary of our assumptions.

Assumptions

1.) Assume that the form of equation (6.1) is valid for the head geometries to be considered.

2.) Assume that the linear math model for the 3-3 mode ferroelectric ceramic longitudinal resonator is valid over the range of frequencies to be considered.

3.) When scaling up or down to a new operating frequency, assume that all linear dimensions are varied proportionately by the same scaling factor, which is defined in equation (3.2).

Scaling equations for fixed-input design.

1.) At some ω_o operating frequency, a transducer element has been thoroughly studied and optimally designed with respect to areas, lengths, power-handling capability, field limitations, etc., and is completely understood over the frequency operating band centered at ω_o.

2.) A new element design is now required at some new center operating frequency, ω_s, which is related to ω_o by a scaling factor, s (Eqn. 3.2).

3.) With only the available information from computer runs and so forth, the designer can scale the existent element in such a manner as to preserve optimization for the most part and he can immediately determine the operation of this scaled element at the new center frequency, ω_s.

4.) To do this, he need only proceed in the following manner: Returning to equation (7.1), we note that, for any scaling factor, s, the values of quantities of interest for the scaled version of an element are related to the original values in the following fashion:

$$s = \frac{\omega_s}{\omega_o} \qquad (8.0)$$

Input voltage, $E_s = E_o$ (8.1)

" current, $I_s = I_o$ (8.2)

" impedance, $Z_s^{in} = Z_o^{in}$ (8.3)

Head velocity, $\frac{(v_r)_s}{s} = \frac{(v_r)_o}{1}$ or $(v_r)_s = s(v_r)_o$ (8.4)

Head Force, $(F_r)_s \times s = (F_r)_o \times 1$ or $(F_r)_s = \dfrac{(F_r)_o}{s}$ (8.5)

Radiation impedance, $(Z_r)_s = \dfrac{(F_r)_s}{(v_r)_s} = \dfrac{(F_r)_o}{s^2 (v_r)_o} = \dfrac{(Z_r)_o}{s^2}$ (8.6)

Input power, $P_s^{in} = E_s I_s \cos \angle Z_s^{in} = P_o^{in}$ (8.7)

Radiated power, $P_s^r = (F_r)_s (v_r)_s \cos \angle (Z_r)_s = P_o{}^r$ (8.8)

Field, $\mathcal{E}_s = \dfrac{E_s}{\ell_s} = s\dfrac{E_o}{\ell_o} = s\mathcal{E}_o$ (8.9)

Strain, $S_s = k(v_s)_s = sS_o$ (8.10)

Radiated power per unit area, $\dfrac{P_s^r}{A_s} = s^2 \dfrac{P_o^r}{A_o}$ (8.11)

Field normalized to radiated power, $\dfrac{\mathcal{E}_s}{\sqrt{P_s^r}} = s \dfrac{\mathcal{E}_o}{\sqrt{P_s^r}}$ (8.12)

etc.

Scaling equations for field-limited design

Further elaboration is possible, of course, but the above equations provide adequate example. Another interesting method for viewing these quantities vs. scaling of frequency is to hold some other performance variable fixed (other than E or I) such as field when field-limited or strain when strain-limited and view the corresponding behavior of the other variables.

For example, consider the following field-limited case:

Assume field, $\mathcal{E}$, to be limited to a maximum of $\mathcal{E}_o$.

Then: $\mathcal{E}_s = \mathcal{E}_o$ (9.1)

Thus, $E_s = \ell_s \mathcal{E}_s = \dfrac{\mathcal{E}_o \ell_o}{s} = E_o/s$ (9.2)

and $\begin{bmatrix} sE_s \\ sI_s \end{bmatrix} = \begin{bmatrix} (A^o K^o), B^o \\ (C^o K^o), D^o \end{bmatrix} \begin{bmatrix} v_r \\ v_r \end{bmatrix}$ from (7.1) (9.3)

$$Is = Io/s \tag{9.4}$$

$$(v_r)_s = (v_r)_o \tag{9.5}$$

$$(F_r)_s = (F_r)_o/s^2 \tag{9.6}$$

$$Z_s^{IN} = \frac{E_s}{I_s} = Z_o^{IN} \tag{9.7}$$

$$P_s^{IN} = EsIs\ \cos LZ^{IN} = \frac{E_o I_o}{s^2}\ \cos LZ^{IN} = \frac{P_o^{IN}}{s^2} \tag{9.8}$$

$$(Z_r)_s = \frac{(Z_r)_o}{s^2} \tag{9.9}$$

etc.